Study of the Effects of Pressure-Induced Viscosity Changes on the Fluid Flow Characteristics in a Circular Tube

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Nomenclature

K_p = fluid compressibility factor, in.^{2}/lb
L = length of pressure release tube, in.
p = fluid pressure, lb/in.^{2}
P_g = pressure in pressure pot at time equal to zero, lb/in.^{2}
P_a = atmospheric pressure, lb/in.^{2}
Q = volume rate of fluid flow, in.^{3}/sec
r, z = radial and longitudinal coordinates, in.
R_o = internal radius of the pressure release tube, in.
t = event time, msec
V_1 = volume of fluid in reservoir, in.^{3}
V_o = volume of fluid that exits the reservoir during depressurization, in.^{3}
w = longitudinal velocity of fluid within bore of pressure release tube, in./sec
θ = time constant, msec
μ = coefficient of viscosity of fluid, lb-sec/in.^{2}
ν = kinematic viscosity of fluid, in.^{2}/sec
ρ = mass density of fluid, slugs/in.^{3}

Theme

A METHOD of determining the response of fluids under high pressure (up to 50,000 psi) to quick-release into the atmosphere is presented in this report. By assuming constant viscosity of the fluid a closed-form solution was obtained. The assumption of constant viscosity was found to give acceptable results for the test fluids up to an initial pressure of approximately 10,000 psi. A numerical analysis became necessary for initial pressures over 10,000 psi, where the pressure-induced viscosity change of the fluid had to be considered.

The experimental work was conducted by observing the flow of a fluid from a pressurized (up to 50,000 psi) reservoir through a circular tube to the atmosphere. The fluid was released to the atmosphere by means of a quick release valve. As the fluid progresses from the reservoir to the atmosphere, the pressure decreases and creates a condition of variable viscosity along the tube length.

The purpose of this study is to find an analytical solution which gives realistic pressure, time, and velocity information for the flow of a variable viscosity fluid through a circular tube.

Contents

A. Experimental Efforts: The experimental work was designed such that there existed three independent variables. They were 1) length of the pressure release tube, 2) type of oil used, and 3) initial pressure in the pressure pot (reservoir).

There were three lengths of exit tubes, all standard 0.062 in. i.d. by 0.250 in. o.d. high-pressure tubing. The lengths of the tubes were 36 in., 72 in., and 144 in.

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Two types of oil samples were used, one a paraffinic base oil and the other a naphthenic base oil. These oils were supplied by the Sun Oil Company and are designated as R-820-53 and R-820-54, respectively.

The reservoir pressure was monitored by pressure cells located within the reservoir. The pressure decay in the reservoir due to quick-release through circular tubes to the atmosphere was found to be of the exponential form

$$P = P_g e^{-t/\theta} \tag{1}$$

where P and P_g are the current and initial reservoir pressures, respectively, t is the event time, and θ is a time constant.

The purpose of the analytical efforts is to derive a theoretical expression for the time constant θ in order that the pressure decay in the reservoir and the flow characteristics in the release tube can be predicted for a given set of conditions.

B. Analytical Efforts: If the conditions of constant temperature, negligible body forces per unit mass, and constant density are invoked, and assuming the fluid flow through the pressure release tube is in the z direction only, the Navier-Stokes equations for a viscous, incompressible fluid reduce to

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{2}$$

At the maximum operating pressure, the viscosity of the fluids used in the experimental program is seven times its atmospheric value whereas the fluid volume is only 13% less. Thus, the condition of incompressibility was assumed.

From Ref. 1, the pressure along the longitudinal axis of the pressure release tube can be considered to vary linearly from the maximum value at the chamber entrance to atmospheric pressure at the end of the tube. The pressure (P) within the tube can be expressed in terms of time and the z coordinate as

$$P = P_g e^{-t/\theta}(1 - z/L) + P_a \tag{3}$$

Substitution of Eq. (3) into Eq. (2) yields

$$\nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) - \frac{\partial w}{\partial t} = -\frac{P_g}{\rho L} e^{-t/\theta} \tag{4}$$

The solution of Eq. (14), assuming for the moment that the fluid viscosity (ν) is unaffected by pressure, and subject to the boundary condition

$$w(R_o, t) = 0 \qquad t \geqq 0 \tag{5}$$

and initial condition

$$w(r, 0) = 0 \qquad 0 < r < R_o \tag{6}$$

is

$$w(r,t) = \frac{2P_o}{\rho L R_o} \sum_{j=1}^{\infty} \frac{J_o(\lambda_j r)}{\lambda_j J_1(\lambda_j R_o)} \left[\frac{1}{1/\theta - \nu\lambda_j^2}\right] [e^{-\lambda_j^2 \nu t} - e^{-t/\theta}] \tag{7}$$

Calculating the volume rate of fluid (Q) passing through the tube in time t, we get

$$Q = \int_0^A w(r,t)\, da \tag{8}$$

or

$$Q = \frac{4\pi P_g}{\rho L} \sum_{j=1}^{\infty} \left[\frac{1}{1/\theta - \nu\lambda_j^2}\right] [e^{-\lambda_j^2 \nu t} - e^{-t/\theta}] \left[\frac{1}{\lambda_j^2}\right] \tag{9}$$

The volume of fluid (V_0) that must exit the chamber during depressurization is calculated by the equation

$$V_o = \int_0^\infty Q\,dt \tag{10}$$

which when integrated and simplified becomes

$$V_o = \frac{4\pi P_g \theta}{\mu L} \sum_{j=1}^{\infty} \frac{1}{\lambda_j^4} \tag{11}$$

The coefficients λ_j are defined as the roots of the equation

$$J_o(\lambda_j R_o) = 0 \tag{12}$$

in order that the boundary conditions, Eq. (5), be identically satisfied. Thus, Eq. (11) can now be written as

$$V_o \cong \pi\theta P_g R_o^4/8\mu L \tag{13}$$

From Ref. 2, the compressibility of the fluid can be expressed by the equation

$$V_o = V_1 P_g K_p \tag{14}$$

where V_1 is the volume of pressurized fluid contained in the reservoir and the fluid compressibility factor (K_p) is

$$K_p = \left[\frac{4.31\times10^{-6}-(6.5\times10^{-11})P_g+(5.03\times10^{-16})P_g^2}{1.0-(4.31\times10^{-6})P_g+(6.5\times10^{-11})P_g^2-(5.03\times10^{-16})P_g^3}\right]$$

Thus, the time constant θ in Eq. (13) can be expressed as

$$\theta = 8\mu L V_1 K_p/\pi R_o^4 \tag{15}$$

Once the system parameters μ, L, V_1, K_p, and R_o are given, Eq. (15) defines the time constant θ that, when used with Eq. (1), determines the pressure decay in a pressure pot that is instantly exposed to the atmosphere via a circular tube. The corresponding flow characteristics in the circular tube are described by the velocity profiles, Eq. (7), and the volume flow rate, Eq. (9). It should be noted that the application of Eq. (15) is limited to fluids that are insensitive to pressure environments, or to pressure levels that create small changes in the fluid viscosity. Experimentation has indicated that, for the subject fluids, Eq. (15) is invalid for pressures in excess of 10,000 psi.

C. Numerical Analysis: The numerical analysis was based on the use of a series of constant pressure increments to simulate the pressure-time trace. Each increment, or step, of time is assumed to have constant viscosity. Thus the working equation for each step is Eq. (7).

For every increment of pressure, the viscosity (v) and the time constant (θ) were calculated for eleven radial positions, beginning at the center-line and progressing out to the edge of the tube.

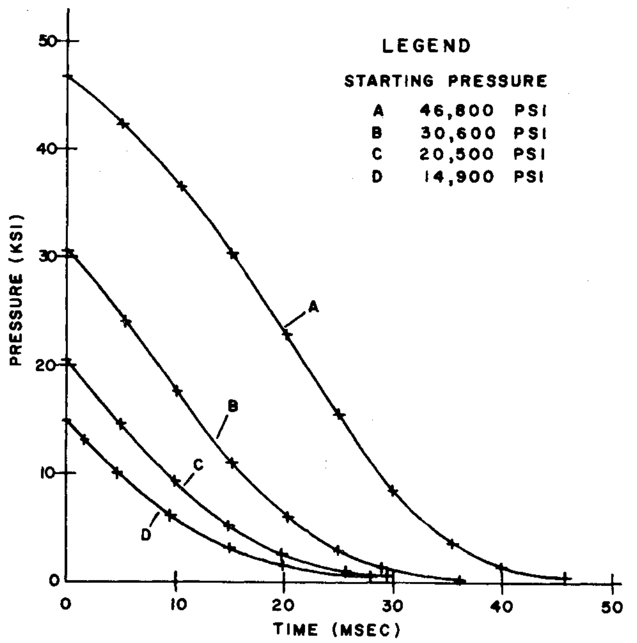

Fig. 1 Pressure-time traces.

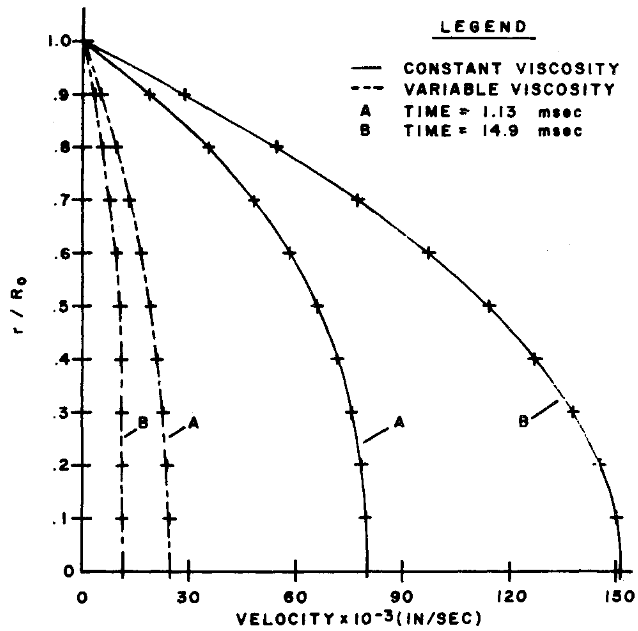

Fig. 2 Comparison of velocity profiles (initial pressure of 30,600 psi).

The pressure was incremented depending on the present value of the pressure. Since the pressure decay in the pressure pot was exponential a smaller increment of pressure was necessitated at the beginning of the run due to the greater slope.

According to the data of Hersey in Ref. 3 and Winer in Ref. 4, the kinematic viscosity can be closely approximated in the pressure range of 0–50,000 psi by the relation

$$v = v_o e^{mP} = v_o e^{mP_g e^{-t/\theta}} \tag{16}$$

where v_o = kinematic viscosity under atmospheric pressure and m = slope of log viscosity-pressure curve.

The viscosity of the various increments was calculated by using Eq. (16). The time constant theta was next calculated by use of Eq. (15).

A computer program was written for use on the IBM 360-50 computer to solve for the instantaneous velocity distribution in the tube for given system parameters.

D. Numerical Results: The results of the computer program are presented in graphical form. Figure 1 shows the decay of pressure in the pressure pot with respect to time. Since the pressure was assumed to decay exponentially over each increment, it could be expected that the total pressure-time trace in Fig. 1 would also have an appreciable decay rate. Figure 2 shows velocity profile traces for constant and variable viscosities for a given initial pressure. It is interesting to note that the effects of variable viscosity may cause a factor of term difference in calculated centerline velocities. The magnitudes of the velocities from the variable viscosity solutions were lower than those obtained by the constant viscosity solution in all cases. Although the velocities become closer as the pressure decreases, the large differences that exist at the high-pressure levels certainly dictate that the effect of pressure on the fluid viscosity must be included in the analysis.

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